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Energy Absorption in Polymeric Foams. II. Prediction of Impact Behavior from Instron Data for Foams with Rate-Dependent Modulus

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Synopsis

Impact behavior can be predicted for rate-dependent foams from constant rate of strain response. The response must be factorized into a rate-dependent modulus function and a strain-dependent function. In this way the rate-dependent modulus can vary throughout the impact as the velocity of the impacting object decreases.

INTRODUCTION

As has been shown in a previous paper,[1] the behavior of certain cellular materials during impact loading can be predicted from stress–strain data obtained at constant strain rates. For some foams, the stress–strain curve is independent of the strain rates at which they are compressed. On the other hand, the mechanical properties of many foams vary with the rate of strain, i.e., some of them exhibit rate-independent and others rate-dependent behavior.

This rate dependence can be due to various factors. If the bulk material is in the transition region between its glassy and rubbery state (Fig. 1), its modulus is strongly rate dependent.[2] In the glassy and rubber plateau regions, however, the bulk material properties are approximately rate independent. The rate dependence can also be caused by parameters related to the structure of the foam. Air passing through small pores will create rate-dependent pneumatic damping.[3] Compression of gases in closed cells[4,5] as well as rupture of closed cell walls may also cause rate dependence.

The material studied previously[1] was a sample of a reticulated polyurethane foam (Scott Paper Co). The large, open cells and the low glass transition temperature of −45°C of the matrix material[6] made this foam an ideal model with approximately rate-independent properties.

This same foam can be modified by coating its structure with bulk materials exhibiting properties different from the urethane itself. Building model foams in this manner permits the change in bulk material properties without affecting seriously the geometry of the structure, which conveniently exhibits no rate-dependent effects. In impact tests, the rate of

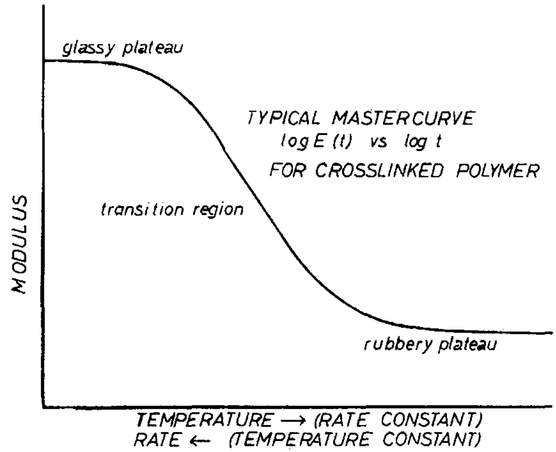

Fig. 1. Typical mastercurve of modulus vs. rate and temperature for crosslinked polymer.

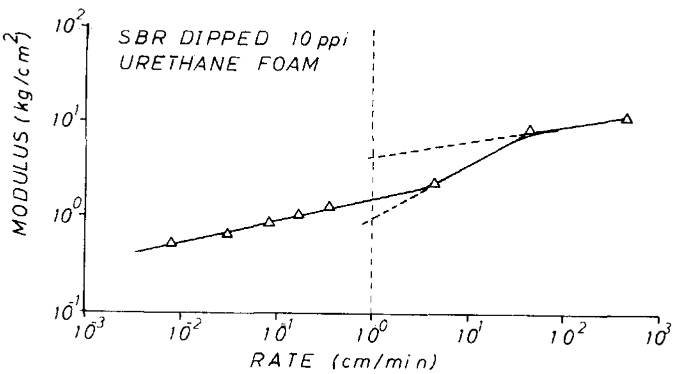

Fig. 2. Modulus vs. rate of strain for SBR-coated urethane foam.

strain varies from the velocity of impact to zero. The analysis of the impact behavior of a rate-independent foam does not have to take into account this changing rate. On the other hand, an analysis of the mechanics of impact for rate-dependent materials must take into account the changing stiffness of the foam, which increases with rate of strain (Fig. 2).

Prediction of impact behavior of cellular materials requires stress–strain, and hysteresis data during compression up to different strains, at different rates of strain. These data can be used to estimate the kinetic energy to deform a material in impact and the energy loss from that deformation.

THEORETICAL CONSIDERATIONS

The energy of deformation E_d of a material is measured as the area under the stress–strain curve,[1]

$$E_d = \int_0^\epsilon \sigma d\epsilon \tag{1}$$

where σ is the loading stress at any strain ϵ. This value is the same as the kinetic energy lost by a freely moving body striking and compressing a sample if all other energy losses are assumed to be negligible:

$$\frac{1}{2}\, mv_0^2 - \frac{1}{2}\, mv_\epsilon^2 = \int_0^\epsilon \sigma d\epsilon \tag{2}$$

where m is the mass of the moving body, v_0 is its velocity immediately before impact, and v_ϵ is its velocity at strain ϵ. This energy balance can be used to determine the velocity profile after an impact.

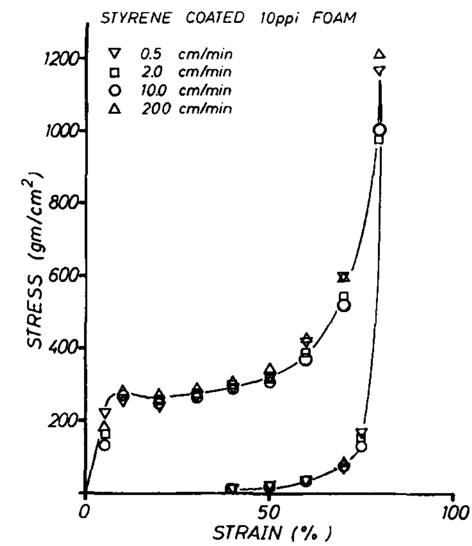

Fig. 3. Stress–strain behavior of styrene-coated urethane foam.

The velocity profiles for rate-independent materials are given by[1]

$$v_\epsilon = \frac{2}{m}\left\{\frac{1}{2}mv_1^2 - \int_0^\epsilon \sigma(\epsilon)d\epsilon\right\}^{1/2},$$

the function $\sigma(\epsilon)$ being independent of rate.

The velocity profile of recovery can be predicted in a similar manner by integrating the unloading stress h from the maximum strain to any strain ϵ as described previously[1]:

$$\frac{1}{2}mv_\epsilon^2 = \int_{\epsilon_{\max}}^{\epsilon} hd\epsilon. \tag{3}$$

For a rate-independent material, the stress is a function of strain alone (Fig. 3),

$$\sigma = f(\epsilon),$$

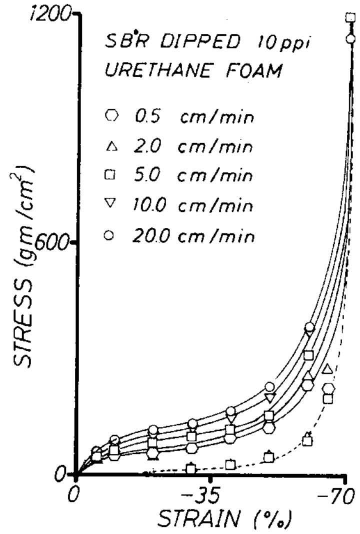

Fig. 4. Stress–strain behavior of SBR-coated urethane foam at various rates of strain.

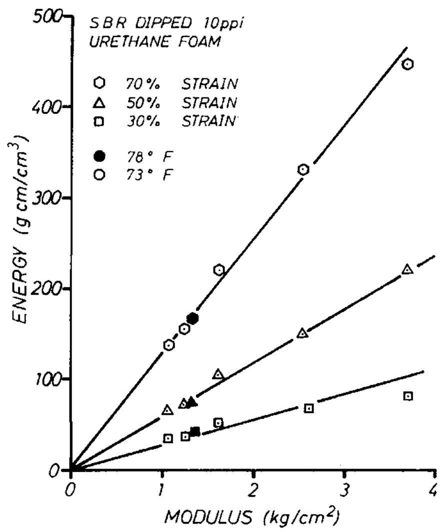

Fig. 5. Strain energy vs. modulus for SBR-coated urethane foam.

but for a rate-dependent material, the stress is a function of the rate of strain as well as the strain (Fig. 4):

$$\sigma = f(v,\epsilon).$$

The energy to compress an SBR-coated sample is proportional to the apparent Young's modulus (slope of stress–strain curve for $\epsilon \to 0$) of the material in the specific conditions of each test (Fig. 5) as discussed by Gent and Thomas.[7]

The loading stress for the rate-dependent material can be normalized with respect to the initial modulus, so that a rate-independent strain function (Fig. 6) can be found:

$$\frac{\sigma}{E_0} = f(\epsilon).$$

For these materials, the rate- as well as strain-dependent stress can therefore be represented by the product of two functions: one which depends on rate alone, $E_0(v)$, and another one, which is independent of rate but depends on strain, $\sigma/E_0(\epsilon)$:

$$\sigma = E_0(v)\left[\frac{\sigma}{E_0}(\epsilon)\right].$$

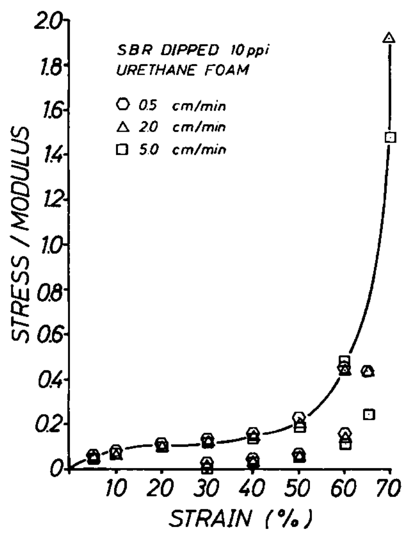

Fig. 6. Stress/modulus vs. strain for SBR-coated urethane foam.

Both $E_0(v)$ and $\sigma/E_0(\epsilon)$ can be determined experimentally and represented by empirical equations. This factorization of the modulus from the stress function has been applied to the deformation of foams[9] as well as to bulk elastomers.[10]

Experimental values of $E_0(v)$ for the SBR-coated polyurethane foam versus the rate of compression are given in Figure 2. Over limited regions of rate, the modulus can be represented by the empirical equation

$$E_0(v) = \kappa v^n$$

where κ is the value of $E_0(v)$ for some arbitrary rate and the slopes of the straight line sections.

The rate-independent function $f(\epsilon)$, as previously described, was originally fitted to the equation proposed by Rusch,[11]

$$f(\epsilon) = \frac{\sigma}{E_0} = \epsilon\psi(\epsilon),$$

where $\psi(\epsilon) = m\epsilon^{-n} + r\epsilon^{+s}$.

The agreement of this representation with the experimental results was not accurate enough for predicting a correct velocity profile. The function $f(\epsilon)$ was therefore determined by least-squares analysis with the help of a computer program developed by Klein[12] to fit the form

$$f(\epsilon) = \sum_{n=0}^{10} B_n\epsilon^n. \tag{4}$$

Applying this empirical function to the energy eq. (2), an equation for the velocity profile during impact of rate dependent materials can be developed as follows:

$$^1/_2\, mv^2 = {}^1/_2 mv_0^2 - \int_0^\epsilon E(v)f(\epsilon)d\epsilon$$

$$^1/_2\, mv^2 = {}^1/_2 mv_0^2 - \int_0^\epsilon Kv^a \left[\sum_{n=0}^{10} B_n\epsilon^n\right] d_\epsilon$$

$$v = \left\{v_0^{2-a} - \frac{K}{m}(2-a)\int_0^\epsilon \left[\sum_{n=0}^{10} B_n\epsilon^n\right] d_\epsilon\right\}^{\frac{1}{2-a}}. \tag{5}$$

For the special case of rate-independent behavior ($a = 0$), these equations reduce to the previously reported ones.[1]

EXPERIMENTAL

Impact and constant-strain-rate loading tests were performed on commercially available reticulated polyurethane foams (10 pores per inch) supplied by Scott Paper Co. The foam was dipped into different liquids to coat the urethane interstices with materials of different bulk properties. Although the thickness of the interstices is changed by this process, the basic structure of the foam (open, large cells) is not greatly effected, i.e., the bulk properties are changed without introducing extraneous means of energy dissipation.

Urethane foam disks (7.5-cm diameter, 2.0 cm thick) were immersed in a 10% solution of polystyrene (Shell 303, $T_g = 100°C$)[13] in benzene for 2 min. They were dried at room temperature and were periodically rotated to assure a uniform polymer deposition throughout the sample. The drying was continued until a constant sample weight was reached. A dry film of approximately 0.1 mm was deposited on the interstices. These samples were used to predict energy absorption of a material in the glassy state (Figs. 1 and 3). Furthermore, the urethane foam, pretreated with a coagulating agent (ammonium nitrate), was dipped into an SBR latex compound (Firestone FR-S 151). The thickness of the SBR coating was approximately 0.3 mm. The 59% styrene SBR ($T_g \approx 0°C$)[14] latex is at room temperature a strongly rate-dependent material (Figs. 2 and 4). These latex-dipped samples were cured at room temperature for one week. They were post-cured for 2 hr at 100°C.

A table model Instron tester was used to obtain stress–strain curves at different constant deformation rates (0.5–20.0 cm/min). A Plasticon tester was used to obtain stress–strain data at higher rates of strain (up to 10000 in./min). All tests were performed at room temperature.

Impact behavior was determined with the help of a modified Scott pendulum as reported previously.[1] The kinetic energy of the pendulum at impact is determined by

$$E_d = {}^1/_2\, m v_{in}^2$$

where v_{in} is the velocity of the center of gravity of the pendulum immediately before impact. Similarly one obtains the energy loss during impact E_h,

$$E_h = {}^1/_2\, m(v_{in}^2 - v_{out}^2).$$

The compression behavior of the styrene-dipped sample was analyzed in the same manner as the one of the untreated urethane foam. The impact data could be related directly to the deformation energy (area under the loading curve) and the hysteresis (area between the loading and unloading curves) of the Instron data.[1] The analysis of the SBR-coated sample is more complicated, since this material proved to be strongly rate dependent (Fig. 2). This rate dependence limits the direct comparison of data obtained at one strain rate with the impact data, since in this test the rate of the deformation decreases with penetration distance.

RESULTS

Stress–strain data obtained for constant rate of compression samples of polystyrene-coated foam are shown in Figure 3. These data were obtained up to a maximum strain at different rates of compression between 0.5 and 20.0 cm/min. The stress–strain curve is not affected by the rate of compression, indicating that the response of the matrix material is rate independent.

Presented in Figure 4 are data describing the compression behavior of the SBR-coated foam deformed at constant rates of strain similar to the method used with the polystyrene-coated foam. There is an obvious effect of strain rate on the stress–strain curve with increasing rates—the stiffness of the foam increases. The recovery stress–strain curve $h(\epsilon)$, however, surprisingly appears to be independent of the rate of deformation.

The impact data were calculated from the knowledge of the pendulum mass m and velocity v. Distance-versus-time values were recorded on a storage oscilloscope and corrected for nonlinearities of the LVDT. The velocity is calculated as the slope of the distance-versus-time plot at any time. The maximum penetration of the hammer can easily be read from the oscilloscope curves. The energy to deform the styrene-dipped samples

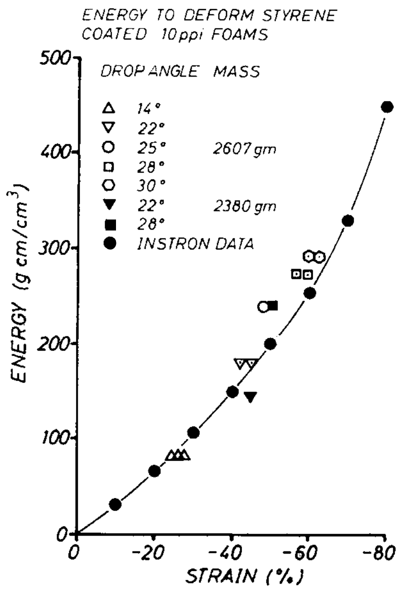

Fig. 7. Strain energy vs. strain for styrene-coated urethane foam.

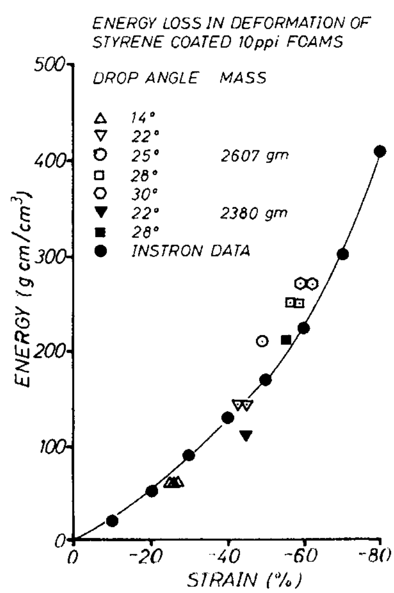

Fig. 8. Hysteresis vs. strain for styrene-coated urethane foam.

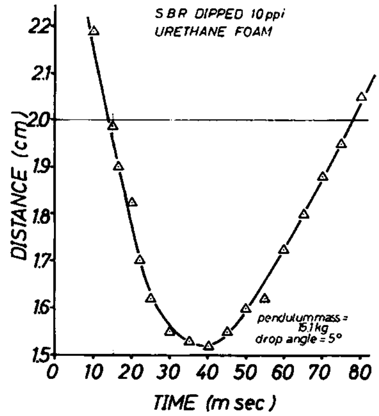

Fig. 9. Oscilloscope tracing (distance vs. time) of an impact test.

to their maximum strain, obtained by Instron data, is compared to E_d of impact in Figure 7. The hysteresis during constant strain rate tests is compared to the impact energy loss E_d in Figure 8.

The behavior of a rate-dependent material cannot be illustrated in the same manner, since variations of the kinetic energy affects the material properties. It was most convenient to determine a velocity profile of the impact, since a velocity profile could be obtained from the oscilloscope data (Fig. 9) and the kinetic energy as well as the material properties are velocity dependent.

If the function log E versus log v were linear over the entire range of velocities studied, eq. (5) would be sufficient to describe the velocity profile of any impact. A linear relation was not found, however, for the SBR-coated materials. The velocity profile could be represented, however, by a series of straight lines over the total range of velocity (Fig. 2, Table I).

TABLE I
$E(v)$ for SBR-Coated Foam

v, cm/sec	E_0, kg/cm²
0–4.0	$1.5v^{0.248}$
4.0–44	$0.86v^{0.569}$
44.0–440	$4.15v^{0.236}$

The velocity profile and maximum penetration can be predicted if the velocity and mass of the impacting object as well as the size, modulus function, and strain function of the foam are known.

The recovery velocity profile is obtained by integrating the recovery stress behavior which is seen, in Figure 4, to be rate independent, eq. (3).

The velocity profile of the impact test is obtained by plotting the slopes of the oscilloscope tracing (distance versus time) versus distance (Fig. 9).

The predicted profile as well as the impact velocity profile are presented in Figures 10 and 11. Figure 10 shows the behavior of a hammer impact-

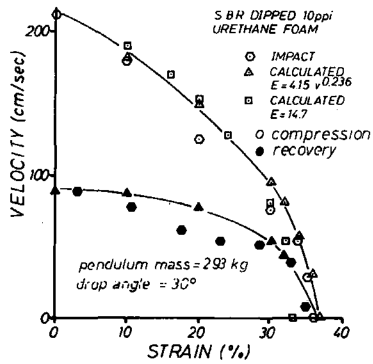

Fig. 10. Velocity profile for impact of SBR-coated urethane foam; v_0 = 228 cm/sec, m = 2.93 kg.

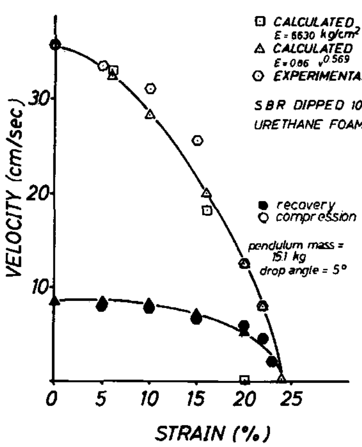

Fig. 11. Velocity profile for impact of SBR-coated urethane foam; v_0 = 34 cm/sec, m = 15.1 kg.

ing at v = 228 cm/sec ($E = 4.15v^{0.236}$ kg/cm²), and Figure 11, one at 34 cm/sec ($E = 0.86v^{0.569}$).

DISCUSSION OF RESULTS

The Instron data in Figure 3 indicate that the modulus of the polystyrene-coated foam is independent of rate at room temperature for the velocity range covered. As was determined for the uncoated urethane system,[1] the stress–strain behavior of these foams will be the same for other loading histories, when the deformation rate varies throughout the test.

The data in Figure 4 indicate a rate-dependent modulus for the SBR-coated material. This indicates a complex function of modulus, and strain is necessary to describe the impact behavior of these foams.

The stored energy and the energy loss data from impact of the glassy material are compared to data calculated from Instron tracings in Figures 7 and 8. The scatter in these data is due primarily to variations of the properties of different samples. The error in measuring the Instron data with a planimeter[1] was eliminated by using the integrated function of $f(\epsilon)$ as described by eqs. (1) and (4).

These calculations were programmed to be computed at strain increments of 2%. The maximum strain, where $\nu_\epsilon = 0$, of an impact test can be estimated by applying eq. (5) if the mass and the velocity are known. The return velocity profile cannot be predicted using the same equation. Since the return stress–strain behavior appeared to be independent of rate and only dependent on maximum strain, the return velocities were calculated graphically, in a manner similar to the calculation of return velocities for rate-independent materials using eq. (3).

The function of the rate dependence is very critical in the analysis of the behavior of rate-dependent foams. It would be more accurate to apply a continuous function describing this dependence than the series of straight lines.

A simplified approach would be to assume a rate-independent modulus which would give the same result as the modulus function. It can be seen from Figure 10 that if the maximum constant value of the expected modulus, $E = 14.7$ kg/cm^2 (at 228 cm/sec), is used rather than the rate-dependent one, a close approximation of the velocity profile is obtained. It is possible that at these higher strain rates the material is approaching glassy behavior, so that such a large part of the energy is absorbed at the high rates shortly after impact that the energy losses at low rates are insignificant.

This approximation is not as successful for a lower velocity impact. In a test with an impact velocity of 34 cm/sec, the modulus at the rate (6.63 kg/cm^2) has been applied as a rate-independent modulus for the entire compression. The velocity profile, however, is not accurately predicted. The greater slope of this rate region (Fig. 2) indicates a higher dependence of modulus on rate. A modulus assumed to be constant in this region leads to significant errors. The application of an appropriate average modulus, on the other hand, would be as tedious as the solution of eq. (5) including the rate dependence.

A continuous function of the entire modulus–rate behavior would increase the accuracy of the analysis. Values of modulus at high rates of strain are difficult to obtain, for high-rate testers are not readily available. Even if these data were easily obtained, the mathematical solution to the problem using a continuous function of E_0 would be complex and not easily integrated.

The experimentation was affected detrimentally by certain problems, as shown in Figure 1; materials whose properties are dependent are very temperature dependent. The rate-dependent samples should be tested at a constant temperature. The temperature could be controlled in the Instron apparatus, but room temperature had to be tolerated in impact studies. Another problem was sample uniformity. It was possible to use the SBR-dipped foams repeatedly. If these samples are heated above the glass transition (ca. 100°C) after compression and cooled to room temperature for 1 hr, no change in stress–strain properties is found. On the other hand, the compression of glassy materials is a destructive process, and samples had to be discarded after each test. Since the dipping process is not easily controlled and the thickness of the polymer coating varies slightly, variations in sample properties are unavoidable.

CONCLUSIONS

It has been shown that impact energy-absorbing characteristics can be predicted for open cell foams by using constant rate of compression data. For rate-independent materials, this behavior can be predicted directly from integrating Instron stress–strain curves. The behavior of rate-dependent materials must be characterized by a rather more complicated function consisting of a rate-independent strain factor and a rate-dependent modulus factor.

If the modulus factor is expressed as an exponential function of rate, the solution to the problem is simple and the predicted impact behavior is reasonable. A more precise description of modulus rate dependence would probably yield more accurate results, but the mathematical solution would be complex.

This analysis has been restricted to materials which loose energy by viscoelastic effects only of the polymer matrix. It is possible to dissipate energy by other rate-dependent mechanisms such as pneumatic damping of small cell material and irreversible compression of closed cell foams. The application of the described method to commercially available energy-absorbing foams is presently being studied.

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